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GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION
SPONSORED PROJECT INITIATION

Date: November 18, 1976

Project Title: Finite Difference Solutions to Problems of Rapid Crack Propogation

Project No: E-23-624

Project Director: Dr. Wilton W. King

Sponsor: Lockheed-Georgia Company

Agreement Period: From 1/1/77 Until 4/15/77

Type Agreement: Purchase Order No. CN65356

Amount: $1,350

Reports Required: Final Report

Sponsor Contact Person (s):

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Defense Priority Rating: None

Assigned to: Engineering Science and Mechanics (School/Laboratory)

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SPONSORED PROJECT TERMINATION

Date: August 5, 1977

Project Title: Method of Finite Differences Applied to 2-D Problems of Rapid Crack Propagation

Project No: E-23-624

Project Director: Dr. Wilton W. King

Sponsor: Lockheed-Georgia Company

Effective Termination Date: 4/15/77

Clearance of Accounting Charges: 4/15/77

Grant/Contract Closeout Actions Remaining:

- X Final Invoice ~~and Closing Documents~~
- _ Final Fiscal Report
- _ Final Report of Inventions
- _ Govt. Property Inventory & Related Certificate
- _ Classified Material Certificate
- _ Other ______

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L 2C-321

May 2, 1977

Mr. R. W. Milling
Departmaent 72-76
Lockheed-Georgia Company
Marietta, GA 30063

Dear Mr. Milling:

Enclosed is one copy of the M. S. Special Problem Report in which Mr. F. P. Hagan summarizes the results of his work supported by Lockheed-Georgia during the period 1 January 1977 to 15 April 1977 under Purchase Order No. CN65356.

Your support of this effort is most appreciated.

Sincerely yours,

Wilton W. King
Associate Professor

/sam

enclosure

A COMPUTER PROGRAM FOR FINITE-DIFFERENCE ANALYSIS OF DYNAMIC FRACTURE PROBLEMS

by

Frank Paul Hagan

A report submitted to Dr. W. W. King in partial fulfillment of the requirements for the degree Master of Science in Engineering Science and Mechanics.

March 1977

Abstract

Finite difference schemes have been widely utilized to generate solutions to both static and dynamic problems in continuum mechanics. Of recent interest has been the application of this method to problems in fracture mechanics. This work considers plane strain elastic behavior of cracked sheets. A generalized FORTRAN program capable of handling arbitrary time independent displacement and stress loading along the boundaries of a finite sheet has been established. The program can easily be extended to examine the problem of a running crack. The program has been applied to a fixed grip edge cracked sheet. Using an extremely coarse mesh, a stress intensity factor 3.8% higher than the theoretical solution for a semi-infinite sheet was obtained. The dynamic problem of an instantaneously appearing crack of finite length was also examined.

I. Introduction

Many two dimensional dynamic fracture and wave propagation problems have been treated using finite difference techniques. The first problem chosen to test our finite difference computer program was an edge cracked sheet with fixed grips. The top of the beam was given a uniform displacement perpendicular to the crack axis.

Shmuely and Peretz [1] solved a DCB problem for fixed grips specifying the displacement of only two interior mesh points. Alterman, Burridge and Lowenthal [2] and Shmuely and Alterman [3] investigated a suddenly appearing crack of finite length, while Stöckl and Auer [4] examined a crack of zero initial length growing with constant velocity.

Two dimensional elastic fracture was examined using a "Lagrangian Finite Difference Analog" by Hanson and Sanford [5], [6]. This technique is described by Petschek and Hanson [7].

Finite difference schemes were employed by Ottaviani [8], Alterman and Lowenthal [9] and Alterman and Rotenberg [10] to examine the propagation of seismic waves in quarter and three quarter planes.

The finite difference scheme used to solve our problem parallels to some extent the procedure followed by Shmuely and Peretz [1].

II. Theoretical Background

Elastic Equations and Boundary Conditions

Nomenclature:

λ, μ	=	Lamé constants
c_1	=	dilatational wave velocity
c_2	=	shear wave velocity
ρ	=	density
σ_{ij}	=	stress tensor
u	=	displacement in x direction
v	=	displacement in y direction

For linearly elastic two-dimensional plane strain problems the stress-strain relations are:

$$\begin{aligned} \sigma_{xx} &= (\lambda + 2\mu)\frac{\partial u}{\partial x} + \lambda\frac{\partial v}{\partial y} \\ \sigma_{yy} &= \lambda\frac{\partial u}{\partial x} + (\lambda + 2\mu)\frac{\partial v}{\partial y} \\ \sigma_{xy} &= \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \end{aligned} \tag{1}$$

The equations of motion are:

$$\begin{aligned} \rho\frac{\partial^2 u}{\partial t^2} &= \frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{xy}}{\partial y} \\ \rho\frac{\partial^2 v}{\partial t^2} &= \frac{\partial\sigma_{xy}}{\partial x} + \frac{\partial\sigma_{yy}}{\partial y} \end{aligned} \tag{2}$$

The Lamé constants are related to the compressional and shear wave velocities by:

$$\begin{aligned} c_1^2 &= \frac{\lambda + 2\mu}{\rho} \\ c_2^2 &= \mu/\rho \end{aligned} \tag{3}$$

Combining (1), (2) and (3) yields:

$$\frac{\partial^2 u}{\partial t^2} = C_1^2 \frac{\partial^2 u}{\partial x^2} + (C_1{}^2 - C_2{}^2) \frac{\partial^2 v}{\partial x \partial y} + C_2{}^2 \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial^2 v}{\partial t^2} = C_2{}^2 \frac{\partial^2 v}{\partial x^2} + (C_1{}^2 - C_2{}^2) \frac{\partial^2 u}{\partial x \partial y} + C_1{}^2 \frac{\partial^2 v}{\partial y^2} \qquad (4)$$

Equations (4) represent the governing differential equations for two-dimensional elastodynamic behavior.

On the boundaries the displacements u and/or v may be specified, or the stresses may be specified. In the latter case equations (1) are utilized. It is convenient to write (1) in the following form.

$$\rho\ \sigma_{xx} = C_1^2 \frac{\partial u}{\partial x} + (C_1^2 - 2C_2^2) \frac{\partial u}{\partial y}$$

$$\rho\ \sigma_{yy} = (C_1^2 - 2C_2^2) \frac{\partial u}{\partial x} + C_1^2 \frac{\partial u}{\partial y} \qquad (5)$$

$$\rho\ \sigma_{xy} = (C_1^2 - 2C_2^2) \left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial x}\right)$$

Finite Difference Scheme

The finite difference scheme followed for the solution of the static crack case is founded upon the method of "Dynamic Relaxation" first described by Day [11]. Essentially, the static problem is attacked as if it were a dynamic problem. The equations of motion are modified by adding an appropriate damping factor proportional to the velocity. Starting with arbitrary initial conditions, the modified equations of motion are integrated using finite difference algorithms in conjunction with the required boundary conditions. As the internal forces tend to equilibrate, the velocity and hence the damping effects vanish. This method is discussed further by Hodgkins [12].

For the two-dimensional elastodynamic problem, the equations of motion become:

$$\theta \frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial t^2} = c_1^2 \frac{\partial^2 u}{\partial x^2} + (c_1^2 - c_2^2) \frac{\partial^2 u}{\partial x \partial y} + c_2^2 \frac{\partial^2 u}{\partial y^2}$$

$$\theta \frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial t^2} = c_2^2 \frac{\partial^2 u}{\partial x^2} + (c_1^2 - c_2^2) \frac{\partial^2 u}{\partial x \partial y} + c_1^2 \frac{\partial^2 v}{\partial y^2} \tag{6}$$

where θ = damping constant.

Since the finite difference operators used for (4) and (6) are the same, one can readily solve a static crack problem thereby obtaining the required initial displacement field for a dynamic problem and then, by setting θ equal to zero, solve the dynamic problems.

Most of the authors mentioned above [1], [3], [4] use the type of finite difference mesh shown in Figure (1).

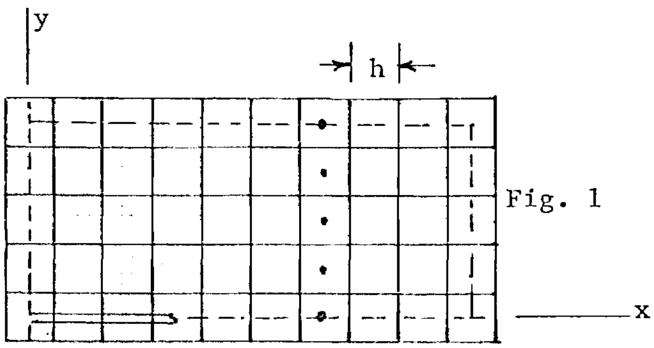


Fig. 1

● material point

— - — boundary

The physical boundaries of the material lie on the x and y axes. Material points lie half way between the grid lines. The grid is extended by four lines at a distance of h/2 past the "real" material boundaries.

This work used the type of mesh shown in Figure (2).

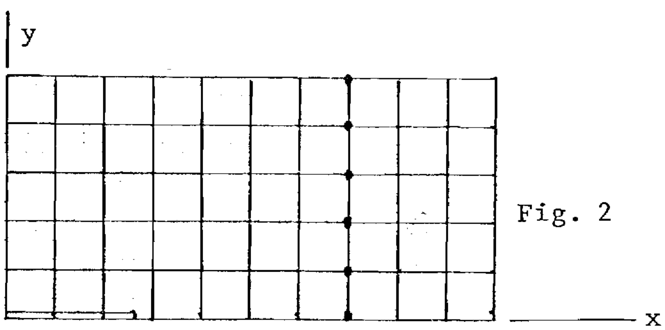


Fig. 2

● material point

The crack is located on the x axis which contains the actual material points, i.e., the material points coincide with the intersection points of the grid. There are no "imaginary" lines extended past the material boundaries. Therefore, the crack propagates along a line of material points in Fig. (2) and between material points in Fig. (1). Note that the crack tip is assumed between two grid lines in both Figs. (1) and (2).

Alterman and Rotenberg [10] discovered that a finite difference scheme utilizing central difference operators for derivatives perpendicular to the boundary lead to numerical instabilities. Therefore, forward difference operators were used for derivatives perpendicular to the boundary. This eliminated the need for the four extra "outside" grid lines [see Fig. 1].

Although a condition of free stress ($\sigma_{yy} = \sigma_{xy} = 0$) should ideally be prescribed along the physical crack boundary, this is not possible using a scheme as shown in Fig. (1). Shmuely and Peretz [1] and Shmuely and Alterman [3] in effect evaluate forward differences perpendicular to the boundary along $y = - h/2$ and central differences parallel to the boundary along $y = \frac{3}{2} h$. The scheme shown in Fig. (2) utilizes forward differences at the crack boundary and central differences at $y = h$.

Assuming the mesh size to be the same in both x and y directions, and denoting the time increment by k, the equations of motion (6) become:

$$\begin{aligned} u(x,y,t+k) = 1/(1 + .5k\theta) \{&2u(x,y,t) - (1 - .5k\theta)\, u(x,y,t-k) + (C_1^2\, k/h)^2 \\ &[u(x+h,y,t) - 2u(x,y,t) + u(x-h,y,t)] + (C_1^2 - C_2^2)\, (k/2h)^2 \\ &[v(x+h,y+h,t) - v(x+h,y-h,t) - v(x-h,y+h,t) + v(x-h,y-h,t)] \\ &+ (C_2 k/h)^2\, [u(x,y,+h,t) - 2u(x,y,t) + u(x,y-h,t)]\} \end{aligned} \tag{8}$$

$$v(x,y,t+k) = 1/(1 + .5k\theta)\ \{2v(x,y,t) - (1 - .5k\theta)\ v(x,y,t-k) + (C_2k/h)^2$$
$$[v(x+h,y,t) - 2v(x,y,t) + v(x-h,y,t)] + (C_1^2 - C_2^2)\ (k/2h)^2$$
$$[u(x+h,y+h,t) - u(x+h,y-h,t) - u(x-h,y+h,t) + u(x-h,y-h,t)$$
$$+ (C_1k/h)^2\ [v(x,y+h,t) - 2v(x,y,t) + v(x,y-h,t)]]\}$$

Note the explicit nature of equations (8). Given the displacements at time t and t-k the displacement at time t+k can be found explicitly.

The possible boundary conditions become:

Along the grid lines x = 0 and x = L;

$$u\left(\begin{matrix}o\\L\end{matrix},y,t\right) = u\left(\begin{matrix}h\\L-h\end{matrix},y,t\right) \pm \left(\frac{C_1^2-2C_2^2}{2C_1^2}\right)\left[v\left(\begin{matrix}h\\L-h\end{matrix},y+h,t\right) - v\left(\begin{matrix}h\\L-h\end{matrix},y-h,t\right)\right]$$

$$- \rho/C_1^2\ \sigma_{xx}(y)$$

$$\tag{9}$$

$$v\left(\begin{matrix}o\\L\end{matrix},y,t\right) = v\left(\begin{matrix}h\\L-h\end{matrix},y,t\right) \pm \frac{1}{2}\left[u\left(\begin{matrix}h\\L-h\end{matrix},y+h,t\right) - u\left(\begin{matrix}h\\L-h\end{matrix},y-h,t\right)\right]$$

$$- \left[\rho/(C_1^2 - 2C_2^2)\right]\ \sigma_{xy}(y)$$

Along the grid lines y = 0 and y = H;

$$u\left(x,\begin{matrix}o\\H\end{matrix},t\right) = u\left(x,\begin{matrix}h\\H-h\end{matrix},t\right) \pm \frac{1}{2}\left[v\left(x+h,\begin{matrix}h\\H-h\end{matrix},t\right) - v\left(x-h,\begin{matrix}h\\H-h\end{matrix},t\right)\right]$$

$$- \left[\ell/(C_1^2 - 2C_2^2)\right]\ \sigma_{xy}(x)$$

$$v\left(x,\begin{matrix}o\\H\end{matrix},t\right) = v\left(x,\begin{matrix}h\\H-h\end{matrix},t\right) \pm \left(\frac{C_1^2-2C_2^2}{2C_1^2}\right)\left[u\left(x+h,\begin{matrix}h\\H-h\end{matrix},t\right)\right.$$

$$- u\left(x - h, \frac{h}{H-h}, t\right)\Big] - \rho / C_1^2 \, \sigma_{yy}(x)$$

Or, of course, the displacements u and v may be specified anywhere on the grid.

An alternate method of satisfying a symmetry condition with respect to a boundary using the difference scheme shown in Fig. (2) is as follows. One can use the equations of motion (8) for mesh points along the boundary in conjunction with the symmetry displacement conditions. This is effectively the same as extending the mesh one spacing beyond the material boundary, and then using the symmetry condition to define the displacements along this "imaginary" line.

The four grid corner displacements pose special problems since they cannot be uniquely determined from the boundary conditions. Alterman and Rotenberg [10] proposed rounding off the corner such that the tangent to the boundary was at an angle of 45° to the coordinate axis. The free stress condition was then applied. Shmuely and Peretz [1] proposed a much simpler solution. The corner displacement was set equal to the average of linear interpolations given along both sides of the corner. Thus, the x and y displacements at the corner shown in Fig. (3) are given by Eq. 10.

$$u(0,0) = u(0,2h) + 2(u(0,h) - u(0,2h)$$
$$= 2u(0,h) - u(0,2h)$$

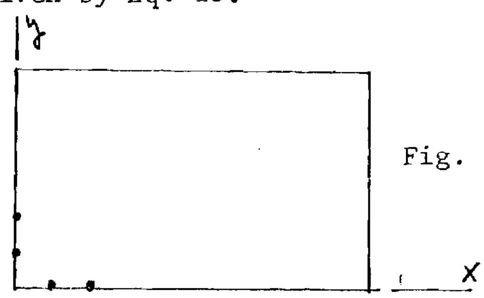


Fig. 3

$$u(0,0) = u(2h,0) + 2(u(h,0) - u(2h,0))$$
$$= 2u(h,0) - u(2h,0)$$

$$(10). \quad u(0,0) = \frac{1}{2}\Big[2u(0,h) + 2u(h,0) - u(0,2h) - u(2h,0)\Big]$$

$$v(0,0) = \frac{1}{2}\Big[2v(0,h) + 2v(h,0) - v(0,2h) - v(2h,0)\Big]$$

Formulas of the form (10) were found to be unsatisfactory when both sides of the corner were free boundaries. In such cases the corner displacement was set equal to the displacement of an adjacent point. Therefore;

$$u(0,0) = u(1,0)$$

$$v(0,0) = v(0,1)$$

The stability of finite difference algorithms has been extensively studied by Richtmeyer [13]. Lax and Richtmeyer [14] examined the convergence of finite difference equations over a finite time interval for increasingly finer meshes. Consequently, their work does not treat the stability of finite difference equations as time $\rightarrow \infty$ for fixed mesh size (fixed Δt). Lewy [15] attacked the stability of the cylindrical wave equation as mesh size decreased.

The stability of the equations of motion (8) with the damping coefficient $\theta \doteq 0$ has been examined by Alterman and Loewenthal [9]. The procedure they followed is basically as follows:

Assume a disturbance in displacement η:

$$\eta = \eta_o \, e^{i(Mha + nhb)} \, e^{ckp}$$

where: $k = \Delta t$

h = grid spacing

m,n,p = integers

$mh = x$

$nh = y$

$kp = t$

$\eta_o = (\eta_1, \eta_2)$, a constant vector

Substituting η into (8) yields two homogeneous equations of the form:

$$\begin{pmatrix} A_1 - A_2 & A_3 \\ A_3 & A_1 - A_4 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

where A_1, A_2, A_3, and A_4 are complicated algebraic functions of K, h, x, y, C_1 and C_2 Non-trivial solution requires

$$\begin{vmatrix} A_1 - A_2 & A_3 \\ A_3 & A_1 - A_4 \end{vmatrix} = 0 \qquad (11)$$

Stability requires $/e^{ck}/ \leq 1$ (12)

Satisfying (11) and (12), and after much algebraic manipulation, the stability condition becomes

$$C_1 \; k \leq h/\left[1 + \frac{C_2^{\;2}}{C_1^{\;2}}\right]^{1/2} \qquad (13)$$

Shmuely and Alterman [16] examined by trial and error the effect of adjusting the dynamic relaxation damping constant θ on the convergence of the solution.

Computer Program

Two three dimensional arrays X DISP(x,y,t) and Y DISP(x,y,t) are needed to store the $\dot{x}$ and y displacements. The first two dimensions of the array refer to the coordinates of the mesh points while the third dimension designate time (t = 2 > t = 1). Since two consecutive time steps are needed to calculate the subsequent node displacements, and since the oldest time step displacement (XDISP(a,b,1)) is only used to calculate the new displacement at its own node, all new displacements when calculated are stored in XDISP (x,y,1). This eliminates the need for a third time step storage space in the displacement array.

The origin of the coordinate axis is labeled (1,1) and for labeling purposes the mesh spacing is assumed to be unity. A measure of the convergence of the solution is given by XCØNV and YCØNV which are defined as:

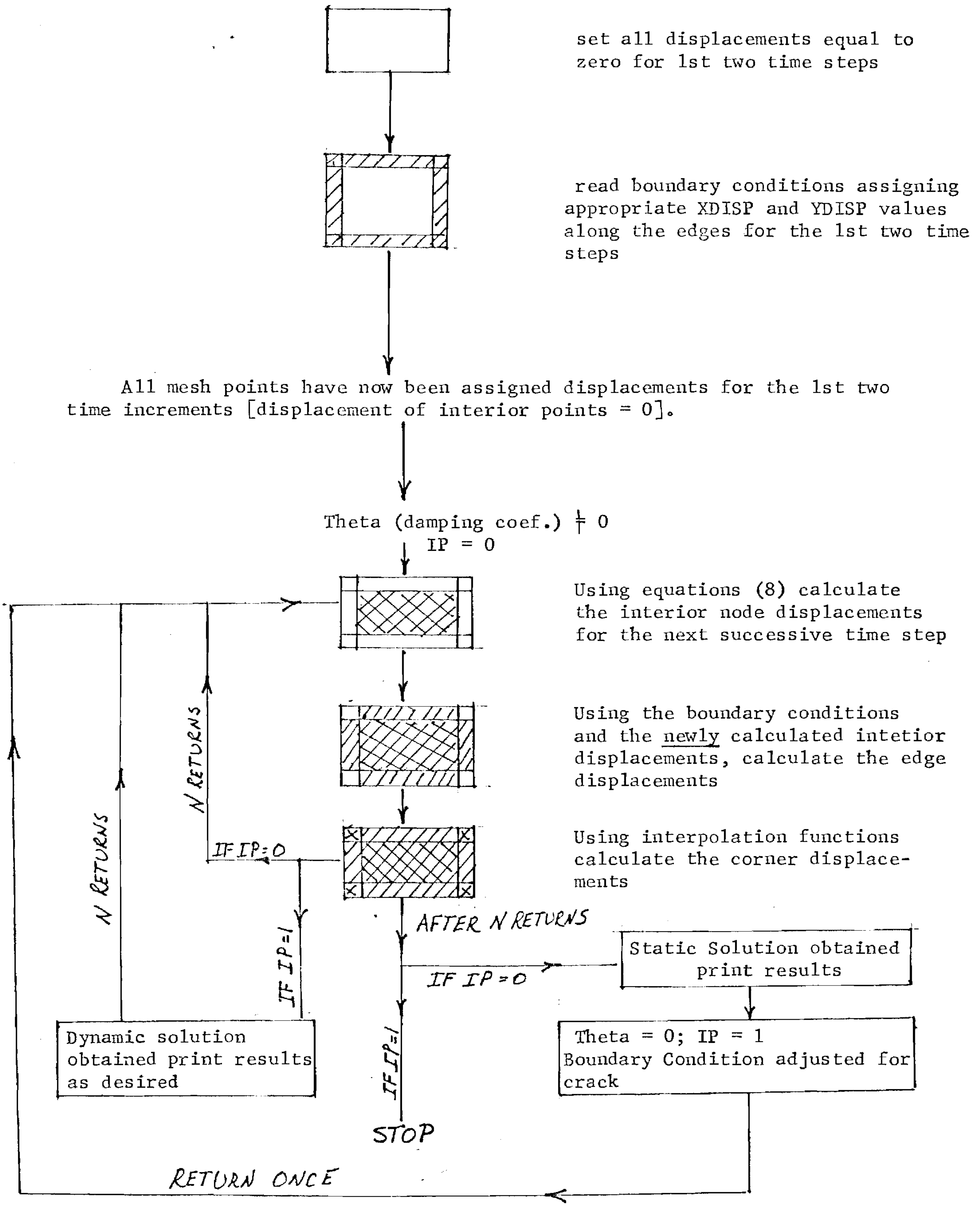
$$\text{XCØNV} = \sum_{I=1}^{w} \sum_{J=1}^{H} [\text{XDISP}(I,J,2) - \text{XDISP}(I,J,1)]^2$$

$$\text{YCØNV} = \sum_{I=1}^{w} \sum_{J=1}^{H} [\text{YDISP}(I,J,2) - \text{YDISP}(I,J,1)]^2$$

where w = width of grid

H = height of grid

On the next page is a crude schematic of the program logic.


set all displacements equal to zero for 1st two time steps
read boundary conditions assigning appropriate XDISP and YDISP values along the edges for the 1st two time steps
All mesh points have now been assigned displacements for the 1st two time increments [displacement of interior points = 0].
Theta (damping coef.) ╪ 0
IP = 0
Using equations (8) calculate the interior node displacements for the next successive time step
Using the boundary conditions and the newly calculated intetior displacements, calculate the edge displacements
Using interpolation functions calculate the corner displacements
N RETURNS
N RETURNS
IF IP=0
IF IP=1
AFTER N RETURNS
Static Solution obtained print results
IF IP=0
Dynamic solution obtained print results as desired
IF IP=1
Theta = 0; IP = 1
Boundary Condition adjusted for crack
STOP
RETURN ONCE

Test Problem

The first problem chosen to test the computer program was an edge cracked sheet under fixed grip loading (see Fig. 4).

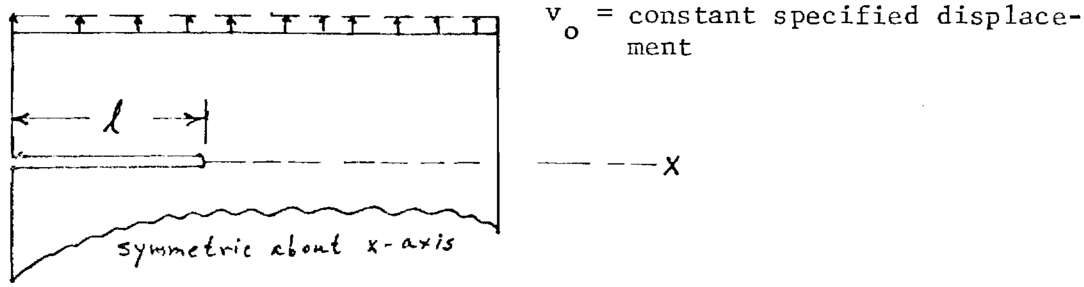


Fig. 4

Using dynamic relaxation, the static solution to the cracked and uncracked problem was obtained. The displacement field of the uncracked problem provided the initial conditions needed for the dynamical problem of a suddenly appearing crack of length ℓ.

The mesh used is shown in Fig. 5. Taking advantage of the symmetry about the x-axis, only the top half of the plane was considered. The following boundary conditions must be satisfied:

$$
\begin{aligned}
&\sigma_{yy} = 0 \text{ and } \sigma_{xy} = 0 && \text{on} && y = 0 \quad \text{for } 0 \le x \le \ell \\
&v = 0 \text{ and } \sigma_{xy} = 0 && \text{on} && y = 0 \quad \text{for } \ell \le x \le w \\
&\sigma_{xx} = 0 \text{ and } \sigma_{xy} = 0 && \text{on} && x = 0, w \text{ for } 0 \le y \le H \\
&v = v_o \text{ and } \sigma_{xy} = 0 && \text{on} && y = H \text{ for } 0 \le x \le w
\end{aligned}
\qquad (14)
$$

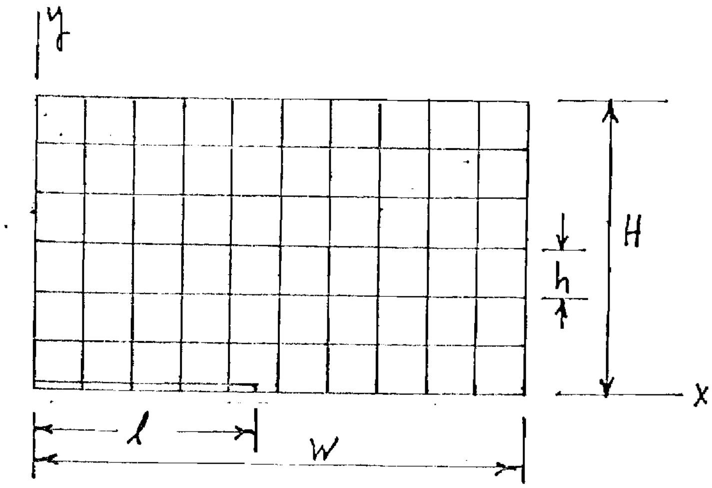


Fig. 5

An 11 x 15 array of mesh points in the y and x directions respectively was used. The crack tip was assumed to lie halfway between the 10th and 11th grid lines on the x axis. The mesh spacing h was assumed to be the same in both directions. The specimen height (H), density (ρ), and dilational wave velocity (C_1) were all assumed equal to one (1). Poissons' ratio ν was assumed equal to 0.25. v_o was taken to be 0.1.

The shear wave velocity is then given by:

$$C_2 = C_1 \left[\frac{1-2\nu}{2(1-\nu)}\right]^{1/2} = 0.57735$$

From equation (13), the maximum allowable time step is:

$$C_1 k = h/\left[1 + \frac{C_2^{\,2}}{C_1^{\,2}}\right]^{1/2}$$

$$k = .1/\left[1 + \frac{(.57735)^2}{1}\right] = .08660$$

For conservatism. K was set equal to .08.

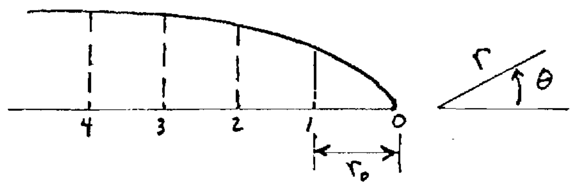
The choice of a suitable damping coefficient is largely a matter of trial and error. Our results tend to substantiate Shmuely and Peretzs' [1] choice of θ = .05/h as yielding the fastest convergence.

As mentioned earlier, the four corner node displacements cannot be obtained from the boundary conditions. For corners at (0,H), (w,H), and (w,0) equations of the form of (10) suffice. However, such a linear interpolation scheme lead to numerical instabilities when applied to the free-free corner. The y displacement at this node was set equal to the y displacement of the adjacent node on the y axis.

Results

The static uncracked plate displacement field is shown in Fig. (6). The static crack boundary displacements are plotted in Fig. (7).

Given the crack surface displacements, the stress intensity factor can be approximated. Near the crack tip the displacements are given by



$$\begin{Bmatrix} u_x \\ u_y \end{Bmatrix} = \frac{K_I}{8G}\left(\frac{2r}{\pi}\right)^{1/2} \begin{Bmatrix} (2K - 1)\cos\theta/2 - \cos 3\theta/2 \\ (2K + 1)\sin\theta/2 - \sin 3\theta/2 \end{Bmatrix}$$

where $K = 3 - 4\nu$ for plane strain.

Solving for K_I in terms of $u_y\big|_{\theta = \pi}$ yields:

$$K_I = \sqrt{\frac{\pi}{2r}}\,\frac{GV}{(1-\nu)} \tag{15}$$

Using two computed data points to fit the curve $a_o + a_1\sqrt{r} + a_2 r$ to the crack boundary displacement and using this approximate displacement function to evaluate (15) yields:

$$K_I = \sqrt{\frac{\pi}{2r_o}}\;\frac{[2V(1) - V(2)]}{(2-\sqrt{2})(1-\nu)}\,G$$

K_I was found to be 0.11075 E.

Nilsson [16] showed the stress intensity factor for an infinite edge cracked strip under identical boundary conditions to be:

$$K_I = \frac{V_o\;\;E}{H^{1/2}(1-\nu^2)}$$

which for our case is 0.10667 E. Thus, our computation resulted in a stress intensity factor 3.8% higher than that of an idealized strip of infinite extent. The y-displacement field for the problem is shown in Fig. (8).

The crack boundary displacement for the problem of an instantaneously appearing crack of finite length is shown in Fig. (9).

X - DISPLACEMENT

.23333E-01	.20000E-01	.16667E-01	.13333E-01	.10000E-01	.66666E-02	.33333E-02	-.[illegible]E-14	-.33333E-02	-.66667E-02
.23334E-01	.20000E-01	.16667E-01	.13333E-01	.99999E-02	.66667E-02	.33333E-02	-.59307E-14	-.33333E-02	-.66667E-02
.23334E-01	.20000E-01	.16667E-01	.13333E-01	.10000E-01	.66667E-02	.33334E-02	-.56172E-14	-.33334E-02	-.66667E-02
.23333E-01	.20000E-01	.16667E-01	.13333E-01	.99997E-02	.66666E-02	.33333E-02	-.67954E-14	-.33333E-02	-.66666E-02
.23333E-01	.20000E-01	.16667E-01	.13333E-01	.10000E-01	.66666E-02	.33332E-02	-.69707E-14	-.33332E-02	-.66666E-02
.23334E-01	.20000E-01	.16667E-01	.13333E-01	.10000E-01	.66667E-02	.33335E-02	-.58041E-14	-.33335E-02	-.66664E-02
.23333E-01	.20000E-01	.16666E-01	.13333E-01	.99998E-02	.66667E-02	.33334E-02	-.64391E-14	-.33334E-02	-.66667E-02
.23333E-01	.20000E-01	.16666E-01	.13333E-01	.99998E-02	.66665E-02	.33332E-02	-.65304E-14	-.33332E-02	-.66665E-02
.23333E-01	.20000E-01	.16667E-01	.13333E-01	.10000E-01	.66667E-02	.33334E-02	-.[illegible]E-14	-.33333E-02	-.66667E-02
.23333E-01	.20000E-01	.16667E-01	.13333E-01	.10000E-01	.66667E-02	.33333E-02	-.71222E-14	-.33333E-02	-.66667E-02
.23333E-01	.20000E-01	.16667E-01	.13333E-01	.10000E-01	.66667E-02	.33333E-02	-.65295E-14	-.33333E-02	-.66667E-02

Y

X

-.10000E-01	-.13333E-01	-.16667E-01	-.20000E-01	-.23333E-01
-.99999E-02	-.13333E-01	-.16667E-01	-.20000E-01	-.23334E-01
-.10000E-01	-.13333E-01	-.16667E-01	-.20000E-01	-.23334E-01
-.99999E-02	-.13333E-01	-.16667E-01	-.20000E-01	-.23333E-01
-.10000E-01	-.13333E-01	-.16667E-01	-.20000E-01	-.23333E-01
-.10000E-01	-.13333E-01	-.16667E-01	-.20000E-01	-.23334E-01
-.99998E-02	-.13333E-01	-.16666E-01	-.20000E-01	-.23333E-01
-.99998E-02	-.13333E-01	-.16666E-01	-.20000E-01	-.23333E-01
-.10000E-01	-.13333E-01	-.16667E-01	-.20000E-01	-.23333E-01
-.10000E-01	-.13333E-01	-.16667E-01	-.20000E-01	-.23333E-01
-.10000E-01	-.13333E-01	-.16667E-01	-.20000E-01	-.23333E-01

Y - DISPLACEMENT

0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01
.20000E-01	.20000E-01	.20000E-01	.20000E-01	.20000E-01	.20000E-01	.20000E-01	.20000E-01	.20000E-01	.20000E-01
.30000E-01	.30000E-01	.30000E-01	.30000E-01	.30000E-01	.30000E-01	.30000E-01	.30000E-01	.30000E-01	.30000E-01
.40000E-01	.40000E-01	.40000E-01	.40000E-01	.40000E-01	.40000E-01	.40000E-01	.40000E-01	.40000E-01	.40000E-01
.50000E-01	.50000E-01	.50000E-01	.50000E-01	.50001E-01	.50001E-01	.50001E-01	.50000E-01	.50001E-01	.50001E-01
.60001E-01	.60001E-01	.60001E-01	.60001E-01	.60001E-01	.60001E-01	.60001E-01	.60001E-01	.60001E-01	.60001E-01
.70000E-01	.70000E-01	.70000E-01	.70000E-01	.70000E-01	.70000E-01	.70000E-01	.70000E-01	.70000E-01	.70000E-01
.80000E-01	.80000E-01	.80000E-01	.80000E-01	.80000E-01	.80001E-01	.80000E-01	.80000E-01	.80000E-01	.80001E-01
.90000E-01	.90000E-01	.90000E-01	.90000E-01	.90000E-01	.90000E-01	.90000E-01	.90000E-01	.90000E-01	.90000E-01
.10000E+00	.10000E+00	.10000E+00	.10000E+00	.10000E+00	.10000E+00	.10000E+00	.10000E+00	.10000E+00	.10000E+00

Y

X

0.	0.	0.	0.	0.
.10000E-01	.10000E-01	.10000E-01	.10000E-01	.10000E-01
.20000E-01	.20000E-01	.20000E-01	.20000E-01	.20000E-01
.30000E-01	.30000E-01	.30000E-01	.30000E-01	.30000E-01
.40000E-01	.40000E-01	.40000E-01	.40000E-01	.40000E-01
.50001E-01	.50000E-01	.50000E-01	.50000E-01	.50000E-01
.60001E-01	.60001E-01	.60001E-01	.60001E-01	.60001E-01
.70000E-01	.70000E-01	.70000E-01	.70000E-01	.70000E-01
.80000E-01	.80000E-01	.80000E-01	.80000E-01	.80000E-01
.90000E-01	.90000E-01	.90000E-01	.90000E-01	.90000E-01
.10000E+00	.10000E+00	.10000E+00	.10000E+00	.10000E+00

.38317E-11 .85347E-11 XCONV, YCONV

figure 6 - Uncracked Sheet

K&E KEUFFEL & ESSER CO. MADE IN U.S.A.

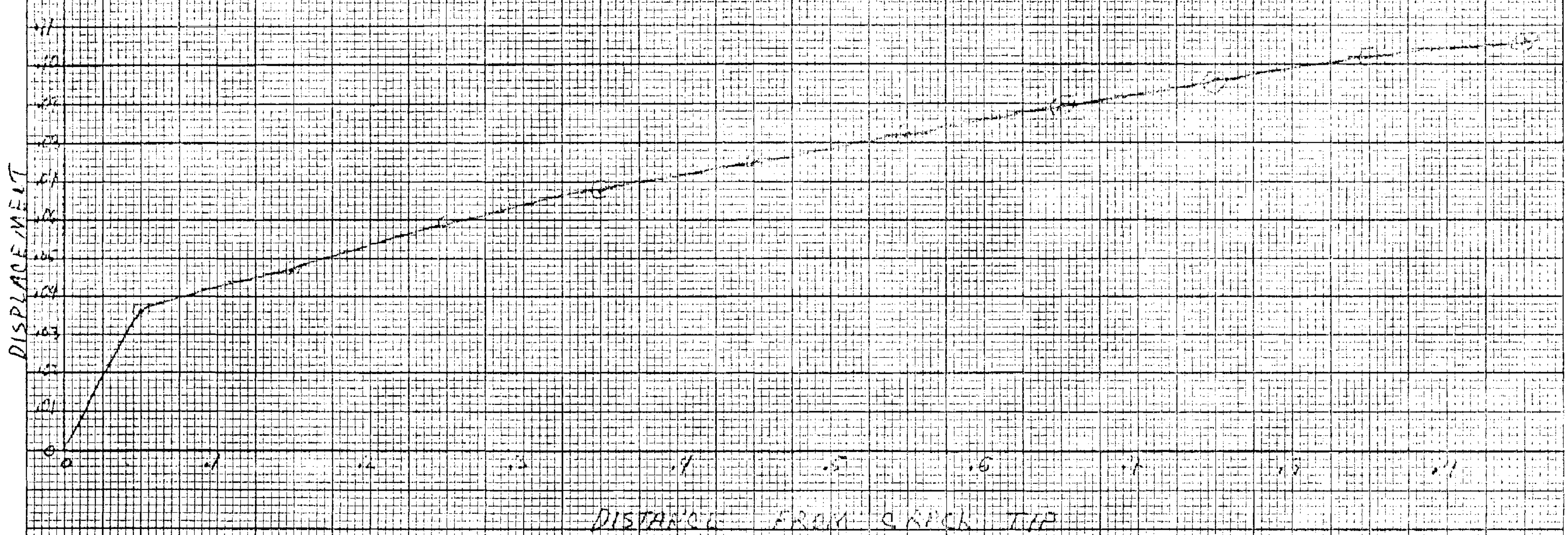


Figure 7 - Y DISPLACEMENT of CRACK SURFACE

Y· DISPLACEMENT

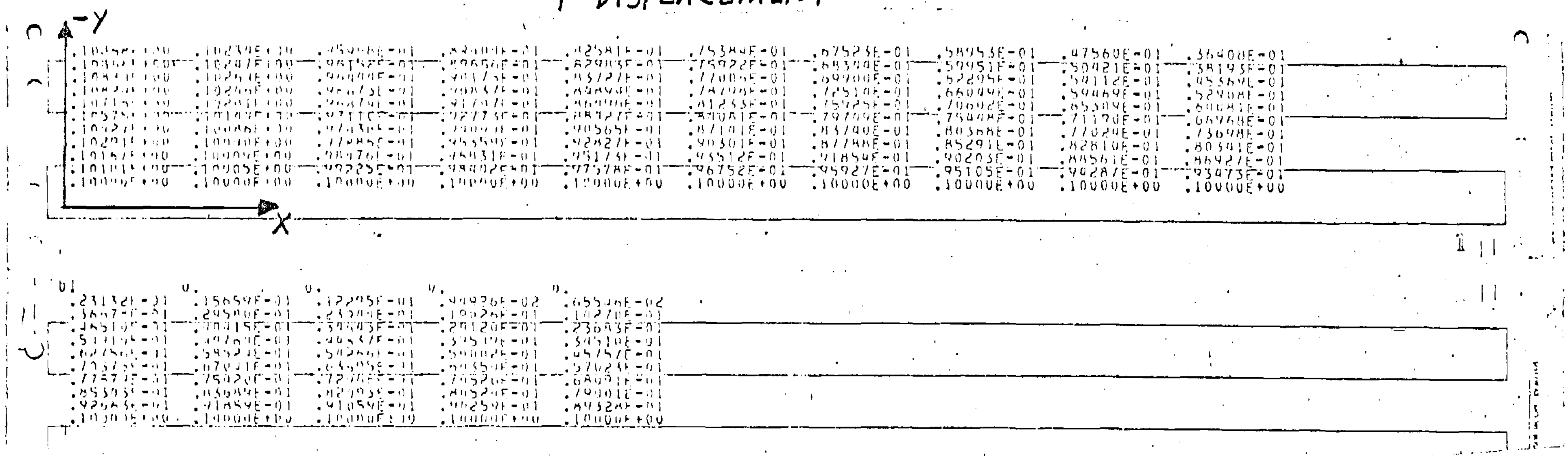
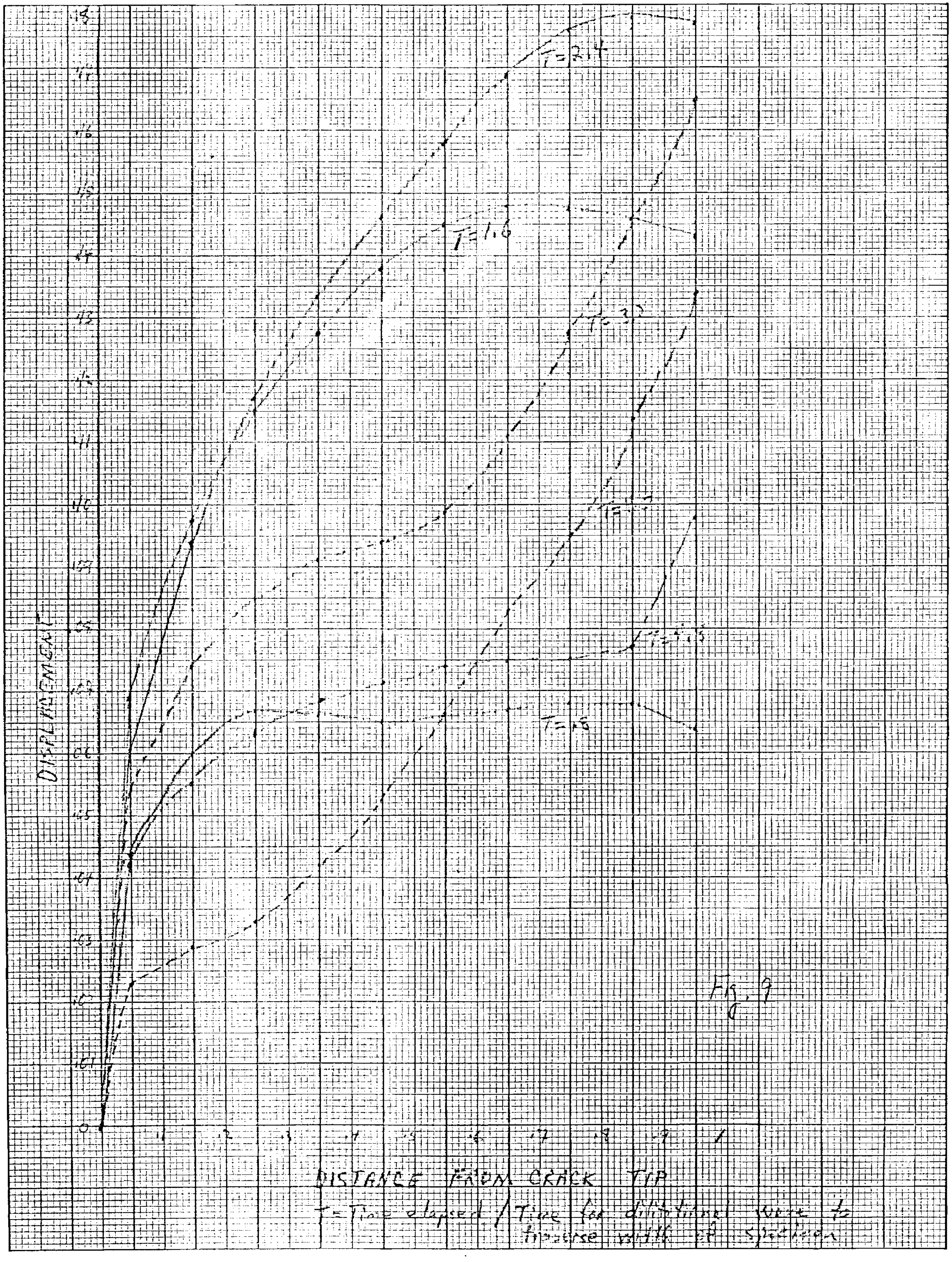


figure 8- Crack Displacements



K+E 10 X 10 TO THE 1/2 INCH KEUFFEL & ESSER CO. MADE IN U.S.A.

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```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION XDISP(15,11,2), YDISP(15,11,2)
      READ(5,*) N1,N2,C1,C2,THETA,TIME,I1,I2,I3,I4,H
      IC=0
      IP=0
      NN=300
      NM1=N1-1
      NM2=N2-1
4     DO 5 I=1,N1
      DO 5 J=1,N2
      XDISP(I,J,1)=0.0
      XDISP(I,J,2)=0.0
      YDISP(I,J,2)=0.0
5     YDISP(I,J,1)=0.0
      IF(I1.NE.1) GO TO 1
      READ(5,*) (XDISP(1,J,2),J=1,N2)
      DO 40 I=1,N2
0     XDISP(1,I,1)=XDISP(1,I,2)
1     IF(I2.NE.1) GO TO 2
      READ(5,*) (XDISP(N1,J,2), J=1,N2)
      DO 41 I=1,N2
1     XDISP(N1,I,1)=XDISP(N1,I,2)
2     IF(I3.NE.1) GO TO 3
      READ(5,*) (YDISP(J,1,2), J=1,N1)
      DO 42 I=1,N1
2     YDISP(I,1,1)=YDISP(I,1,2)
3     IF(I4.NE.1) GO TO 99
      READ(5,*) (YDISP(J,N2,2), J=1,N1)
      DO 43 I=1,N1
3     YDISP(I,N2,1)=YDISP(I,N2,2)
9     DO 100 ICCC=1,NN
      IC=IC+1
      DO 10 I=2,NM1
      DO 10 J=2,NM2
      XDISP(I,J,1)=(1./(1.+.5*TIME*THETA))*(2*XDISP(I,J,2)-(1.-.5*TIME
     A*THETA)*XDISP(I,J,1)+1./(H**2.)*
     2                   ((C1*TIME)**2.)*(XDISP(I+1,J,2)-2.*XDISP(I,J,2)+
     3 XDISP(I-1,J,2))+((C1**2.-C2**2.)*(TIME/2.)**2.)*(YDISP(I+1,J+1,2)
     4 -YDISP(I+1,J-1,2)-YDISP(I-1,J+1,2)+YDISP(I-1,J-1,2))*1./(H**2.)+
     B 1./(H**2.)*
     5 ((C2*TIME)**2.)*(XDISP(I,J+1,2)-2.*XDISP(I,J,2)+XDISP(I,J-1,2)))
      YDISP(I,J,1)=(1./(1.+.5*TIME*THETA))*(2*YDISP(I,J,2)-(1.-.5*TIME
     A*THETA)*YDISP(I,J,1)+1./(H**2.)*
     2                   ((C1*TIME)**2.)*(YDISP(I+1,J,2)-2.*YDISP(I,J,2)+
     3 YDISP(I-1,J,2))+((C1**2.-C2**2.)*(TIME/2.)**2.)*(XDISP(I+1,J+1,2)
     4 -XDISP(I+1,J-1,2)-XDISP(I-1,J+1,2)+XDISP(I-1,J-1,2))*1./(H**2.)+
     B 1./(H**2.)*
     5 ((C2*TIME)**2.)*(YDISP(I,J+1,2)-2.*YDISP(I,J,2)+YDISP(I,J-1,2)))
0     CONTINUE
      IF(I1.NE.2) GO TO 11
      DO 101 J=2,NM2
      XDISP(1,J,1)=XDISP(2,J,2)+(C1**2.-2.*C2**2.)/(2.*C1**2.)*
     2 (YDISP(1,J+1,2)-YDISP(1,J-1,2))
1     YDISP(1,J,1)=YDISP(2,J,2)+.5*(XDISP(1,J+1,2)-XDISP(1,J-1,2))
1     IF(I1.NE.0) GO TO 12
      DO 102 J=2,NM2
      I=1
```

```
      YDISP(I,J,1)=1./(1.+.5*TIME*THETA) *(2.*YDISP(I,J,2)-(1.-.5*TIME)
     2 *YDISP(I,J,1))+2.*(C2*TIME)**2.*(YDISP(I+1,J,2)-YDISP(I,J,2))
     3 +(C1**2.-C2**2.)*(TIME/2.)**2.*(XDISP(I+1,J+1,2)+XDISP(I+1,J-1,2)
     4 )*2.+(C1*TIME)**2.*(YDISP(I,J+1,2)-2.*YDISP(I,J,2)+
     5 YDISP(I,J-1,2))
  102 CONTINUE
   12 IF(I2.NE.2) GO TO 13
      DO 103 J=2,NM2
      XDISP(N1,J,1)=XDISP(NM1,J,2)-(C1**2.-2.*C2**2.)/
     2(2.*C1**2.)*(YDISP(N 1,J+1,2)-YDISP(N 1,J-1,2))
      YDISP(N1,J,1)=YDISP(NM1,J,2)-.5* (XDISP(N 1,J+1,2)-
     2XDISP(N 1,J-1,2))
  103 CONTINUE
   13 IF(I2.NE.0) GO TO 14
      DO 104 J=2,NM2
      I=N1
      YDISP(I,J,1)=1./(1.+.5*TIME*THETA) *(2.*YDISP(I,J,2)-(1.-.5*TIME)
     2 *YDISP(I,J,1))+2.*(C2*TIME)**2.*(YDISP(I+1,J,2)-YDISP(I,J,2))
     3 +(C1**2.-C2**2.)*(TIME/2.)**2.*(XDISP(I+1,J+1,2)+XDISP(I+1,J-1,2)
     4 )*2.+(C1*TIME)**2.*(YDISP(I,J+1,2)-2.*YDISP(I,J,2)+
     5 YDISP(I,J-1,2))
  104 CONTINUE
   14 IF(I3.NE.2) GO TO 15
      DO 105 I=2,NM1
      XDISP(I,1,1)=XDISP(I,2,2)+.5*(YDISP(I+1,1,2)-YDISP (I-1,1,2))
  105 CONTINUE
   15 IF(I3.NE.0) GO TO 16
      DO 106 I=2,NM1
      J=1
      XDISP(I,J,1)=1./(1.+.5*TIME*THETA) *(2.*XDISP(I,J,2)-(1.-.5*TIME)
     2 *XDISP(I,J,1))+2.*(C2*TIME)**2.*(XDISP(I+1,J,2)-XDISP(I,J,2))
     3 +(C1**2.-C2**2.)*(TIME/2.)**2.*(YDISP(I+1,J+1,2)+YDISP(I+1,J-1,2)
     4 )*2.+(C1*TIME)**2.*(XDISP(I,J+1,2)-2.*XDISP(I,J,2)+
     5 XDISP(I,J-1,2))
  106 CONTINUE
   16 IF(I3.NE.3) GO TO 150
      DO 31 I=2,10
      YDISP(I,1,1)=YDISP(I,2,2)+(C1**2.-2.*C2**2.)/(2.*C1**2.)*
     2  (XDISP(I+1,1,2)-XDISP(I-1,1,2))
      XDISP(I,1,1)=XDISP(I,2,2)+.5*(YDISP(I+1,1,2)-YDISP (I-1,1,2))
   31 CONTINUE
      DO 32 I=11,15
      XDISP(I,1,1)=XDISP(I,2,2)+.5*(YDISP(I+1,1,2)-YDISP (I-1,1,2))
   32 CONTINUE
  150 IF(I4.NE.2) GO TO 17
   17 IF(I4.NE.0) GO TO 18
      DO 108 I=2,N1
      J=N2
      XDISP(I,J,1)=1./(1.+.5*TIME*THETA) *(2.*XDISP(I,J,2)-(1.-.5*TIME)
     2 *XDISP(I,J,1))+2.*(C2*TIME)**2.*(XDISP(I+1,J,2)-XDISP(I,J,2))
     3 +(C1**2.-C2**2.)*(TIME/2.)**2.*(YDISP(I+1,J+1,2)+YDISP(I+1,J-1,2)
     4 )*2.+(C1*TIME)**2.*(XDISP(I,J+1,2)-2.*XDISP(I,J,2)+
     5 XDISP(I,J-1,2))
  108 CONTINUE
   18 DO 107 I=2,NM1
      XDISP(I,N2,1)=XDISP(I,NM2,2)-.5*(YDISP(I+1,N 2,2)-YDISP(I-1,N 2,2))
     2 )
```

```
 107 CONTINUE
     XDISP(N1,N2,1)=XDISP(N1-1,N2,2)+XDISP(N1,N2-1,2)-
    2 .5*(XDISP(N1-2,N2,2)+XDISP(N1,N2-2,2))
     XDISP(N1,1,1)=XDISP(N1-1,1,2)+XDISP(N1,2,2)-.5*
    2 (XDISP(N1-2,1,2)+XDISP(N1,3,2))
     XDISP(1,1,1)=XDISP(2,1,2)+XDISP(1,2,2)-.5*(XDISP(3,1,2)+
    2 XDISP(1,3,2))
     XDISP(1,N2,1)=XDISP(2,N2,2)+XDISP(1,N2-1,2)-
    2 .5*(XDISP(3,N2,2)+XDISP(1,N2-2,2))
     DO 55 I=1,N1
     DO 55 J=1,N2
     XTEMP=XDISP(I,J,1)
     XDISP(I,J,1)=XDISP(I,J,2)
     XDISP(I,J,2)=XTEMP
     YTEMP=YDISP(I,J,1)
     YDISP(I,J,1)=YDISP(I,J,2)
  55 YDISP(I,J,2)=YTEMP
     YCONV=0.0
     XCONV=0.0
     DO 65 I=1,N1
     DO 65 J=1,N2
     XCONV=(XDISP(I,J,2)-XDISP(I,J,1))**2.+XCONV
  65 YCONV=(YDISP(I,J,2)-YDISP(I,J,1))**2.+YCONV
     IF(IP.NE.1) GO TO 100
     YDISP(1,1,1)=YDISP(1,2,2)
     WRITE(6,77)
     WRITE(6,305) (YDISP(I,1,2),I=1,10)
 305 FORMAT(1X,10E12.5)
     IC=0
 100 CONTINUE
     WRITE(6,77)
  77 FORMAT(1X,///)
     WRITE(6,88) ((XDISP(I,J,2),I=1,10),J=1,N2)
  88 FORMAT(1X,10E12.5)
     WRITE(6,77)
     WRITE(6,97) ((XDISP(I,J,2),I=11,N1),J=1,N2)
  97 FORMAT(1X,5E12.5)
     WRITE(6,77)
     WRITE(6,88) ((YDISP(I,J,2),I=1,10),J=1,N2)
     WRITE(6,77)
     WRITE(6,97) ((YDISP(I,J,2),I=11,N1),J=1,N2)
     WRITE(6,77)
     WRITE(6,93) XCONV,YCONV
  93 FORMAT(1X,2E12.5)
     THETA=0.
     I3=3
     IP=IP+1
     NN=10
     IF(IP.EQ.1) GO TO 99
  90 STOP
     END
```